Sequence Alignment

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Exercise: Scoring an ungapped alignment

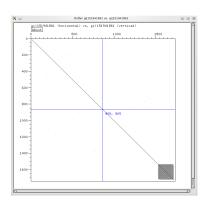
Given two sequences and a scoring matrix, find the offset that yields the best scoring ungapped alignment.

Exercise: Scoring an ungapped alignment

Given two sequences and a scoring matrix, find the offset that yields the best scoring ungapped alignment.

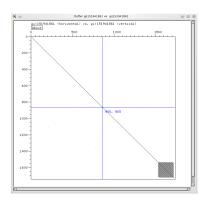
```
def score(S,x,y):
    """Return alignment score for subsequences x and y for scoring
    matrix S (represented as a dict)"""
    assert(len(x) == len(y))
    return sum(S[i][i] for (i,i) in zip(x,y))
def subseqs(x,y,i):
    """ Return subsequences of x and y for offset i."""
    if(i > 0):
       y = y[i:]
    elif(i < 0):
        x = x[-i:]
    L = min(len(x), len(y))
    return \times [:L], \vee [:L]
def ungapped(S.x.v):
    """Return best offset, score, and alignment between sequences x
    and y for scoring matrix S (represented as a dict)."""
    hest = None
    hest score = None
    for i in range(-len(x)+1,len(y)):
        (sx, sy) = subseqs(x, y, i)
        s = score(S.sx.sv)
        if(s > best_score):
            best score = s
            hest = i
    return best, best_score, subseqs(x,y,best)
```

Dotplots



 Unbiased view of all ungapped alignments of two sequences

Dotplots

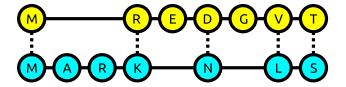


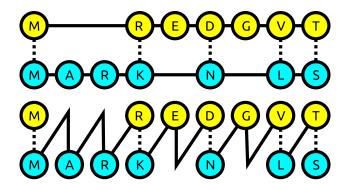
- Unbiased view of all ungapped alignments of two sequences
- Noise can be filtered by applying a smoothing window to the diagonals.

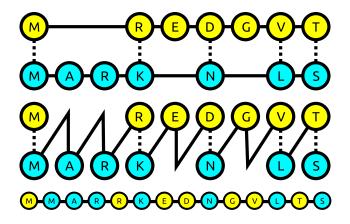
Exercise: Scoring a gapped alignment

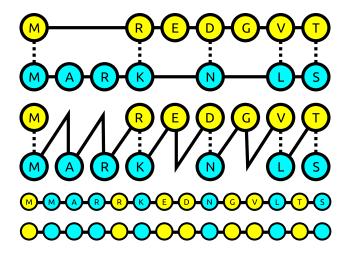
- Given two equal length gapped sequences (where "-" represents a gap) and a scoring matrix, calculate an alignment score with a -1 penalty for each base aligned to a gap.
- ② Write a new scoring function with separate penalties for opening a zero length gap (e.g., G=-11) and extending an open gap by one base (e.g., E=-1).

$$S_{gapped}(x, y) = S(x, y) + \sum_{i}^{gaps} (G + E * len(i))$$













Binomial formula:

$$\binom{k}{r} = \frac{k!}{(k-r)!r!}$$



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Stirling's approximation:

$$x! \approx \sqrt{2\pi} \left(x^{x + \frac{1}{2}} \right) e^{-x}$$



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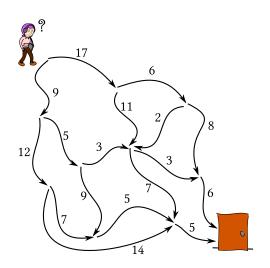
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$$\binom{2n}{n} = \frac{(2n)!}{n!n!}$$

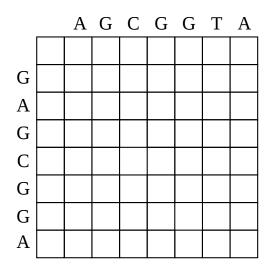
Stirling's approximation:

$$x! \approx \sqrt{2\pi} \left(x^{x+\frac{1}{2}} \right) e^{-x}$$

$$\binom{2n}{n} \approx \frac{2^{2n}}{\sqrt{\pi n}}$$

Dynamic Programming





| | | A | G | C | G | G | T | A |
|---|------------|------------------------------|-------------|----|----|----------------------|----|----|
| | 0. | ₋ -1 ₋ | - 2₄ | 3, | 4_ | _ -5 _ | 6_ | _7 |
| G | <u>-</u> 1 | | | | | | | |
| A | <u>-</u> 2 | | | | | | | |
| G | †3 | | | | | | | |
| C | † -4 | | | | | | | |
| G | <u>-</u> 5 | | | | | | | |
| G | <u></u> -6 | | | | | | | |
| A | <u>^</u> 7 | | | | | | | · |

| | | A | G | C | G | G | T | A |
|---|----------------------------|-------------|-----|------|----|----------------------|----|---|
| | 0 | <u>-1</u> - | A | - 3, | 4. | _ -5 _ | 6_ | 7 |
| G | [↑] 1 | -1 | - (| 3 | | | | |
| A | - | A | A | | | | | |
| G | G | - | G | | | | | |
| C | † -4 | | | | | | | |
| G | <u>-</u> 5 | | | | | | | |
| G | <u></u> 6 | | | | | | | |
| A | <u>↑</u> 7 | | | | | | | |

| | | A | G | C | G | G | T | A |
|---|----------------|----------------------|-------------|----|------------------|------------------------------|----|----|
| | 0 | - -1 - | - 2₄ | 3, | ₋ -4. | ₋ -5 ₋ | 6, | _7 |
| G | †1 | -1 | | | | | | |
| A | <u>-</u> 2 | | | | | | | |
| G | ⁺ 3 | | | | | | | |
| С | [↑] 4 | | | | | | | |
| G | ↑ 5 | | | | | | | |
| G | ^ 6 | | | | | | | |
| A | <u>^</u> 7 | | | | | | | · |

| | | A | G | C | G | G | T | A |
|---|-----------------|-----------|-------------------------|----|------------------------------|------------------------------|----|----|
| | 0 | <u>-1</u> | - 2 _∢ | 3_ | ₋ -4 ₋ | ₋ -5 ₋ | 6, | _7 |
| G | †1 | -1 | 0 | | | | | |
| A | <u>+</u> 2 | | | | | | | |
| G | ¹ 3 | | | | | | | |
| C | [†] -4 | | | | | | | |
| G | ¹ -5 | | | | | | | |
| G | <u></u> 6 | | | | | | | |
| A | <u></u> 7 | | | | | | | |

| | | A | G | C | G | G | T | A |
|---|------------|-----------|-------------|--------------|------------------------------|------------------------------|----|----|
| | 0 | <u>-1</u> | - 2₄ | 3, | ₋ -4 ₋ | ₋ -5 ₋ | 6, | _7 |
| G | †1 | -1 | 0* | - - 1 | | | | |
| A | <u>-</u> 2 | | | | | | | |
| G | <u></u> 3 | | | | | | | |
| C | † -4 | | | | | | | |
| G | <u>-</u> 5 | | | | | | | |
| G | <u></u> -6 | | | | | | | |
| A | <u></u> 7 | | | | | | | |

| | | A | G | C | G | G | T | A |
|---|-----------------|-----------|-------------|-----|----|--------------|----|------------|
| | 0 | <u>-1</u> | - 2₄ | Г 🦹 | , | - -5, | 6_ | <u>-</u> 7 |
| G | †1 | -1 | `0 ≺ | 1* | -2 | | | |
| A | <u>+</u> 2 | | | | | | | |
| G | ¹ 3 | | | | | | | |
| C | † -4 | | | | | | | |
| G | ¹ -5 | | | | | | | |
| G | <u></u> -6 | | | | | | | |
| A | <u></u> 7 | | | | | | | |

| | | A | G | C | G | G | T | A |
|---|------------|-----------|-------------|-------------|----|------------------------------|----|---|
| | 0 | <u>-1</u> | - 2₄ | 3 | -4 | ₋ -5 ₋ | 6, | 7 |
| G | †1 | -1 | 0* | - -1 | -2 | -3 | | |
| A | <u>-</u> 2 | | | | | | | |
| G | <u></u> 3 | | | | | | | |
| C | † -4 | | | | | | | |
| G | <u>-</u> 5 | | | | | | | |
| G | <u></u> 6 | | | | | | | |
| A | <u></u> 7 | | | | | | | |

| | | A | G | C | G | G | T | A |
|---|-----------------|-----------|-------------------------|----|----|------------------------------|-------------|--------------|
| | 0 | <u>-1</u> | - 2 _∢ | 3_ | -4 | ₋ -5 ₋ | 6_ | _7 |
| G | <u>†</u> 1 | -1 | 0~ | 1* | -2 | -3 | -4 - | - - 5 |
| A | <u>+</u> 2 | | | | | | | |
| G | - 3 | | | | | | | |
| C | [†] -4 | | | | | | | |
| G | ¹ -5 | | | | | | | |
| G | <u></u> 6 | | | | | | | |
| A | <u></u> 7 | | | | | | | |

| | | A | G | C | G | G | T | A |
|---|-------------------------------|-----------|-------------------------|------------|----|------------------------------|-----------------|-----------------|
| | 0 | <u>-1</u> | - 2 _∢ | 3_ | -4 | ₋ -5 ₋ | 6, | _7 |
| G | ¹ / ₋ 1 | -1 | 0~ | 1 - | -2 | -3 | -4 * | ⁻ -5 |
| A | ¹ / ₄ 2 | 0 | | | | | | |
| G | ¹ 3 | | | | | | | |
| C | [†] -4 | | | | | | | |
| G | ¹ -5 | | | | | | | |
| G | <u></u> 6 | | | | | | | |
| A | <u></u> 7 | | | | | | | |

| | | A | G | C | G | G | T | A |
|---|-----------------|----------------|----------------------|----------------|----------------|------------------------------|-----------------|--------------|
| | 0 | <u>-1</u> | - -2 √ | 3_ | -4 | ₋ -5 ₋ | 6, | 7 |
| G | ¹ -1 | -1 | 0 | 1 | -2 | -3 | <u>-4</u> | - - 5 |
| A | ¹ 2 | 0 | <u>-</u> 1 | -1 | -2 | 3 | 4 | `-3 |
| G | ⁺ 3 | -1 | 1 | 0 | 0 | 1, | -2 _* | 3 |
| С | [↑] 4 | ⁺ 2 | Ō | 2 | -1 < | -0 | -1• | -2 |
| G | € 5. | _3 | | 1 | 3, | `-2 ∢ | - 1 - | -0 |
| G | ^ 6, | 4 | 2 | -0 | _2 | 4 | 3 | -2 |
| A | <u></u> 7 | -5 | -3 | [†] 1 | [†] 1 | 3 | `3 | 4 |

| | | A | G | C | G | G | T | A |
|---|-----------------------|------------|---------------------|----------------|----------------|----------------|----------------|--------------|
| | 0 | -1 | - -2 <u></u> | 3_ | -4 | 5_ | 6_ | 7 |
| G | $\frac{\uparrow}{4}1$ | -1 | 0< | 1 | -2 | -3 | -4 | - - 5 |
| A | <u>+</u> 2 | 0 | 1 | -1 | -2 | 3_ | -4 | `-3 |
| G | ^T 3 | -1 | 1+ | 0 | 0 - | -1, | -2 | 3 |
| С | - 4 | <u>-</u> 2 | Ō | 2 | ÷1 < | - 0 - | 1 | - -2 |
| G | <u>-</u> 5 | -3 | -1 | 1 | 3, | -2◄ | - 1 - | -0 |
| G | ¹ -6 | - 4 | <u>-</u> 2 | 0 | 2 | 4 | - 3 | - 2 |
| A | 1 7 | 5-5 | - 3 | [†] 1 | [†] 1 | _ 3 | 3 | 4 |

Homework

Implement Needleman-Wunsch global alignment with zero gap opening penalties. Try attacking the problem in this order:

- Initialize and fill in a dynamic programming matrix by hand (e.g., try reproducing the example from my slides on paper).
- Write a function to create the dynamic programming matrix and initialize the first row and column.
- Write a function to fill in the rest of the matrix
- Rewrite the initialize and fill steps to store pointers to the best sub-solution for each cell.
- Write a *backtrace* function to read the optimal alignment from the filled in matrix.

If that isn't enough to keep you occupied, read the dynamic programming references from the class website. Try to articulate in your own words the logic for the speed-ups and trade-offs in the Myers and Miller approach.